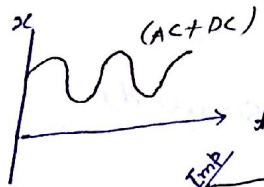
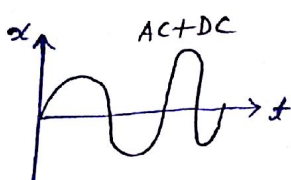
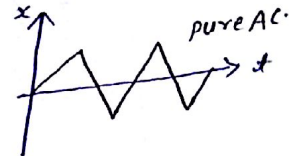
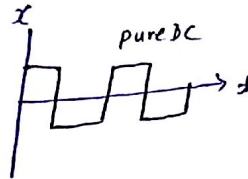
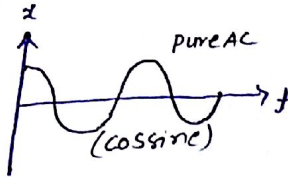
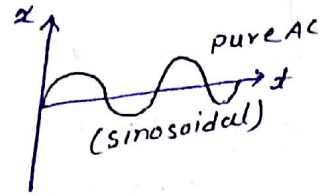
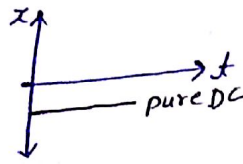
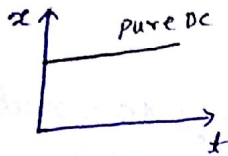


ALTERNATING CURRENT

Electrical signal

→ Direct (unidirectional, ϕ const)

→ Alternating (Bidirectional, periodic, positive peak equal to negative peak position.)



* AC measured by Hot wire instrument
→ Heating Effect of current prove AC.

* DC measured by moving coil galvanometer.
→ proved by magnetic Effect of current.

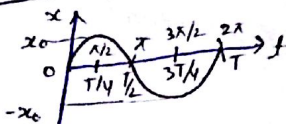
Fundamental Alternating signal.

$$x = x_0 \sin(\omega t \pm \phi)$$

x → Instantaneous value
 x_0 → peak value
 $2x_0$ → peak to peak value
Phase Angle → $\omega t + \phi$
 ω → Angular frequency.

ϕ = phase difference b/w current & voltage.

Case-I $x = x_0 \sin \omega t$

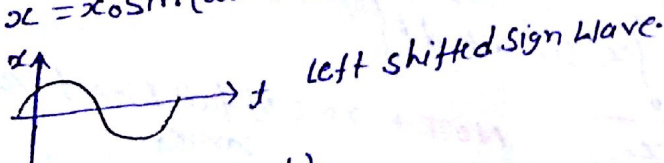


$$2\pi = T$$

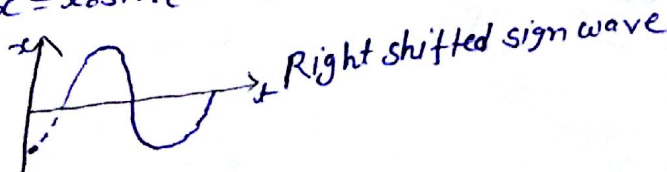
Eg → $\frac{\pi}{3} = \frac{T}{6}$
 $\frac{\pi}{4} = \frac{T}{8}$
 $\frac{\pi}{6} = \frac{T}{12}$

→ In one cycle, direction change twice
→ $0 - T/2$ ⊕ve Half cycle.
→ $T/2 - T$ ⊖ve half cycle.

Case-II $x = x_0 \sin(\omega t + \phi)$

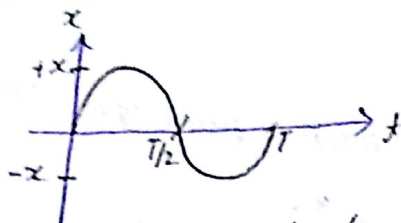


Case-III $x = x_0 \sin(\omega t - \phi)$



Fundamental Alternating signal

$$x = x_0 \sin \omega t$$



11) $\rightarrow$ Average/Mean value/DC voltage

Case-I $\rightarrow$ For complete cycle

$$\begin{aligned}\langle x \rangle &= \frac{1}{T} \int_0^T x dt \\ &= \frac{1}{T} \int_0^T x_0 \sin \omega t dt\end{aligned}$$

$$\boxed{\langle x \rangle_T = 0}$$

NOTE $\rightarrow$ For any AC signal Average for one cycle is always zero.

Case-II $\rightarrow$ For Half cycle $T/2$

$$\begin{aligned}\langle x \rangle_{+1/2} &= \frac{1}{T/2} \int_0^{T/2} x \cdot dt \\ &= \frac{2}{T} \int_0^{T/2} x_0 \sin \omega t dt\end{aligned}$$

$$\langle x \rangle_{+1/2} = \frac{2x_0}{\pi}$$

$$\begin{aligned}\langle x \rangle_{+1/2} &= +\frac{2x_0}{\pi} \\ \langle x \rangle_{-1/2} &= -\frac{2x_0}{\pi}\end{aligned}$$

NOTE $\rightarrow$ For Half cycle Average value for sinusoidal voltage is may be $\oplus$ ve, $\ominus$ ve, zero.

* Average voltage/current

$$\langle V \rangle_{\text{cycle}} = 0$$

$$\langle i \rangle_{\text{cycle}} = 0$$

12) $\rightarrow$ Root Mean Square velocity (rms, Apparent, Effective, virtual)

$$x_{\text{rms}} = \left\{ \frac{1}{T} \int_0^T x^2 dt \right\}^{1/2}$$

$$x_{\text{rms}} = \left\{ \frac{1}{T} \cdot \int_0^T x_0^2 \sin^2 \omega t dt \right\}^{1/2}$$

$$\boxed{x_{\text{rms}} = \frac{x_0}{\sqrt{2}}}$$

NOTE $\rightarrow$ rms of full & Half cycle is same.

POWER LOSS

$$P_{inst} = VI = V_0 \sin(\omega t) \cdot I_0 \sin(\omega t + \phi)$$

$$= \frac{V_0 I_0}{2} [\cos \phi - \cos(2\omega t + \phi)]$$

$$\langle P_{avg} \rangle_{cycle} = \frac{V_0 I_0}{2} \cos \phi = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi$$

$$= V_{rms} \cdot I_{rms} \cdot \cos \phi$$

$$P = I^2 R$$

$$H = I^2 R t$$

NOTE → * If nothing is mentioned then given AC voltage consider rms only.
 * For Heat & power calculation only RMS value is used.
 * 220 volt AC is more dangerous than 220 volt DC b/c peak value of AC is 311. (और 2 अ आ के सटके मरिगी)

Form Factor (FF)

$$FF = \frac{x_{rms}}{x_{average}} = \frac{\frac{x_0}{\sqrt{2}}}{\frac{2x_0}{\pi}} = \boxed{\frac{\pi}{2\sqrt{2}}}$$

NOTE → * $\langle \sin \omega t \rangle_T = 0$ * $\langle \sin^2 \omega t \rangle_T = 1/2$
 * $\langle \cos \omega t \rangle_T = 0$ * $\langle \cos^2 \omega t \rangle_T = 1/2$
 * $\langle \sin^2 \omega t \rangle_T = 0$
 * $\langle \cos^2 \omega t \rangle_T = 0$

EX → $I = I_1 + I_2 \sin \omega t$

Short trick Average for one cycle.
 ① $\langle I \rangle_T = \frac{\langle I_1 + I_2 \sin \omega t \rangle_T}{I_1 + I_2 \times 0} = I_1$

② Rms for one cycle
 $I_{rms} = \left\{ \langle I^2 \rangle_T \right\}^{1/2}$
 $= \left\{ \langle I_1^2 + I_2^2 \sin^2 \omega t + 2 I_1 I_2 \sin \omega t \rangle \right\}^{1/2}$
 $I_{rms} = \left\{ I_1^2 + \frac{I_2^2}{2} + 0 \right\}^{1/2}$
 $= \sqrt{I_1^2 + \frac{I_2^2}{2}}$

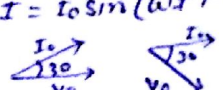
Phase

When a physical parameter changes sinusoidally with sign then it can be represented by an arrowed straight line & this representation is called phase. Where length of line represent peak value.

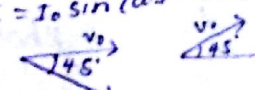
Phase diagram

It is diagram having phase current & voltage both.

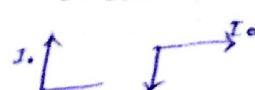
EX → $V = V_0 \sin \omega t$
 $I = I_0 \sin(\omega t + 30^\circ)$



EX → $V = V_0 \sin \omega t$
 $I = I_0 \sin(\omega t - 45^\circ)$



EX → $V = V_0 \sin \omega t$
 $I = I_0 \sin \omega t$

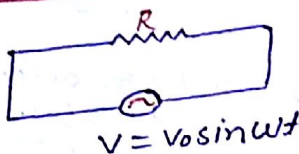


Phase difference (ϕ) $\rightarrow$ It is difference b/w phase of V & I .

$$\text{Power factor} = \cos \phi$$

AC CKTS

1.1 $\rightarrow$ pure Resistive ckt (R)



$$V = V_0 \sin \omega t$$

By KVL at any instant.

$$* I = \frac{V}{R} = \frac{V_0}{R} \sin \omega t$$

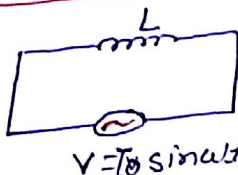
$$I = I_0 \sin \omega t$$

ii $\rightarrow$ phase diagram $\xrightarrow{I_0} \xrightarrow{V_0}$

iii $\rightarrow$ Phase difference = 0

iii $\rightarrow$ power factor $\cos \phi = 1$ (max)
(R consumes the total power of ckt).

1.2 $\rightarrow$ pure Inductive ckt (L)



$$V = V_0 \sin \omega t$$

By KVL

$$* I = I_0 \sin(\omega t - \pi/2)$$

* voltage leading the current
by $\pi/2$

* phase difference = $\pi/2$

* phasor diagram $V_0 \uparrow$ $I_0 \rightarrow$

* power factor = $\cos \phi = 0$

* power = $V_{rms} I_{rms} \cos \phi = 0$

Inductive Reactance (X_L)

$$* X_L = \omega L = 2\pi f L$$

$$* X_L \propto f$$

* unit $\rightarrow$ ohm (R, X_L, X_C)

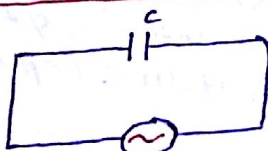
$R \rightarrow$ Resistance = R

$L, C \rightarrow$ Reactance (X) $\begin{matrix} \nearrow X_L \\ \searrow X_C \end{matrix}$

$\Downarrow$
 $Z =$ Impedance

* [General term that include both resistance & reactance.]

1.3 $\rightarrow$ pure capacitive ckt (C)



$$V = V_0 \sin \omega t$$

$$V = \frac{q}{C} = 0$$

$$q = CV$$

$$* I = I_0 \sin \omega t + \pi/2$$

capacitive Reactance (X_C)

$$* X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

* unit $\rightarrow$ ohm

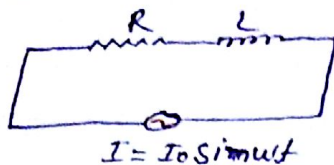
$$* X_C \propto \frac{1}{f}$$



* voltage lagging by current by $\pi/2$ angle
so $\phi = \pi/2$

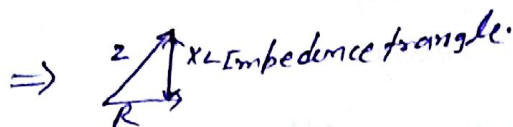
* phase diagram $V_0 \downarrow$ $I_0 \rightarrow$

14) → Series RL →



Let, $I = I_0 \sin \omega t$
 $V_R = V_0 R \sin \omega t$
 $V_L = V_0 L \sin(\omega t + \pi/2)$
 When $\begin{cases} V_0 R = I_0 R \\ V_0 L = X_0 X_L \end{cases}$

phase Diagram



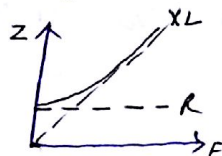
* i) → $V_0 = \sqrt{V_0 R^2 + V_0 L^2}$

* ii) → Impedance (Z)

$Z = \sqrt{R^2 + X_L^2}$

* iii) → unit = ohm

* iv) → Z v/s f



* $f = 0 \Rightarrow X_L = 0 \Rightarrow Z = R$
 $f(\uparrow) \Rightarrow X_L(\uparrow) \Rightarrow Z(\uparrow)$
 $f(\infty) \Rightarrow X_L(\infty) \Rightarrow Z(\infty)$

~~Imp~~ V Leading

* phase difference (ϕ) = Acute Angle

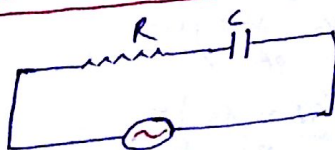
$\phi = \tan^{-1} \left(\frac{V_0 L}{V_0 R} \right) = \tan^{-1} \left(\frac{X_L}{R} \right)$

* power factor = $\cos \phi = \frac{V_0 R}{V_0} = \frac{R}{Z}$

~~Imp~~ Equation of supply voltage

$V_{\text{supply}} = V_0 \sin(\omega t + \phi)$

15) → Series RC



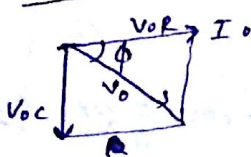
Let, ckt current
 $I = I_0 \sin \omega t$

$V_R = V_0 R \sin \omega t$

$V_C = V_0 C \sin(\omega t - \pi/2)$

When $\begin{cases} V_0 R = I_0 R \\ V_0 C = X_0 C \end{cases}$

Phase diagram



* $V_0 = \sqrt{V_0 R^2 + V_0 C^2}$

* Impedance (Z)

$Z = \sqrt{R^2 + X_C^2}$

* Z v/s f

$\begin{cases} * f = 0 & X_C = \infty \Rightarrow Z = \sqrt{R^2 + X_C^2} = \infty \\ * f(\uparrow) & X_C = \downarrow \Rightarrow Z \downarrow \\ * f = \infty & X_C = 0 \Rightarrow Z = R \end{cases}$



* Voltage Laging.

* Phase difference (ϕ) $\Rightarrow$ Acute Angle.

$$\phi = \tan^{-1} \left(\frac{V_{oc}}{V_{or}} \right) = \tan^{-1} \left(\frac{X_c}{R} \right)$$

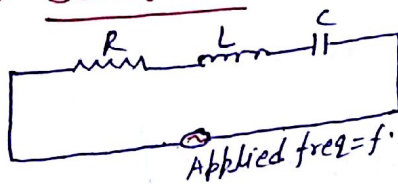
* Power factor

$$PF = \cos \phi = \frac{V_{or}}{V_o} = \frac{R}{Z}$$

* Equation of power supply

$$V_{supply} = V_o \sin(\omega t - \phi)$$

161 $\rightarrow$ Series RLC

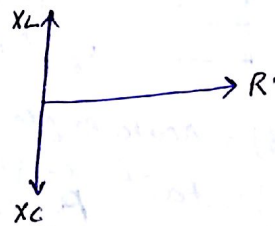
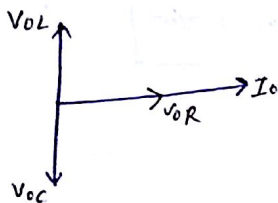


$$\text{Let } I_{ext} = I_o \sin \omega t$$

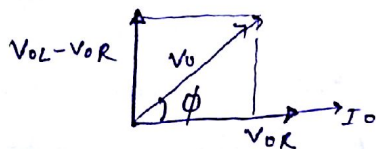
$$V_R = V_{or} \sin \omega t$$

$$V_L = V_{oL} \sin(\omega t + \pi/2)$$

$$V_C = V_{oc} \sin(\omega t - \pi/2)$$



Case-I If $V_{oL} > V_{oc}$



* Series R-L behaviour.

$$* V_o = \sqrt{V_{or}^2 + (V_{oL} - V_{oc})^2}$$

$$* Z = \sqrt{R^2 + (X_L - X_C)^2}$$

* Voltage leading.

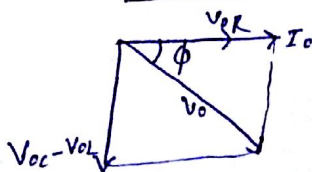
$$* \phi = \tan^{-1} \left(\frac{V_{oL} - V_{oc}}{V_{or}} \right)$$

$$= \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

* power factor

$$P.F = \cos \phi = \frac{V_{or}}{V_o} = \frac{R}{Z}$$

Case-II $\rightarrow$ If $V_{oc} > V_{oL}$



* Series R-C behaviour

$$* V_o = \sqrt{V_{or}^2 + (V_{oc} - V_{oL})^2}$$

$$* Z = \sqrt{R^2 + (X_C - X_L)^2}$$

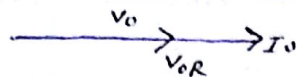
$$* \text{Power factor } \cos \phi = \frac{V_{or}}{V_o} = \frac{R}{Z}$$

* 'V' Laging.

$$* \phi = \tan^{-1} \left(\frac{V_{oc} - V_{oL}}{V_{or}} \right)$$

$$= \tan^{-1} \left(\frac{X_C - X_L}{R} \right)$$

CASE - III → Voltage Resonance ($V_{oL} = V_{oc}$)



- * pure 'R'
- * $V_o = V_{oR}$
- * $\phi = 0$
- * power factor = $\cos \phi = 1$ (Max)
- * $Z = R$ (min)
- * $I = \frac{V}{Z} = \frac{V}{R}$ (Max)

At Resonance

$$\begin{aligned}(X_L)_r &= (X_C)_r \\ \rightarrow (V_{oL})_r &= (V_{oc})_r \\ (I_0 X_L)_r &= (I_0 X_C)_r \\ (\omega L)_r &= \left(\frac{1}{\omega C}\right)_r\end{aligned}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \text{ (Rad/sec)}$$

$$F_r = \frac{1}{2\pi\sqrt{LC}} (X_L)$$

⇒ Z v/s f

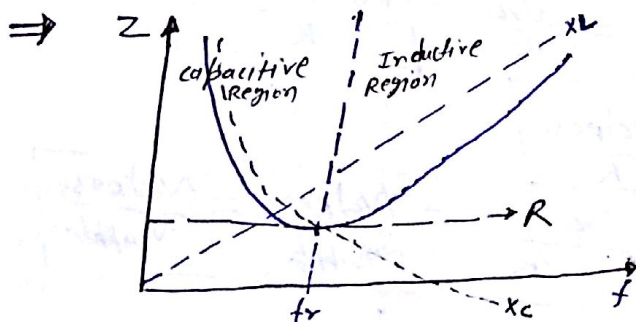
ii) → $f = 0$
 $R = R$
 $X_L = 0$
 $X_C = \infty$
 $Z = \infty = X_C$

iii) → $0 < f < f_r$
 $R = R$
 $X_L \uparrow$
 $X_C \downarrow$
 $X_C > X_L$ } series R-C

iii) → $f = f_r$
 $R = R$
 $X_L = X_C$ } pure 'R'

iv) → $f_r < f < \infty$
 $R = R$
 $X_L > X_C$ } series R-L

v) → $f = \infty$
 $R = R$
 $X_L = \infty$
 $X_C = 0$
 $Z = \infty = X_L$
 pure 'L'



Graph
 $I = V/Z$

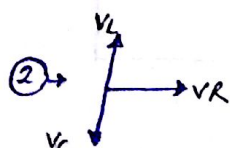
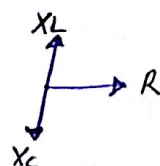


General Knowledge

① → Resistance = R
 Reactance = X

$$\begin{aligned}X_L &= \omega L \\ X_C &= \frac{1}{\omega C}\end{aligned}$$

$$Z \Rightarrow$$

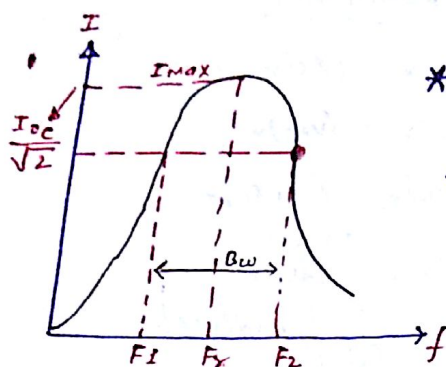


③ → $\phi = \tan^{-1} \left(\frac{V}{R} \right)$

iv) → $\cos \phi = \left(\frac{R}{Z} \right) = \text{power factor.}$

Half power frequency

At this freq. current become $\frac{1}{\sqrt{2}}$ times of I_{max} . So power become Max.



$$* f_1 = f_r - \frac{R}{4\pi L}$$

$$* f_2 = f_r + \frac{R}{4\pi L}$$

* Bandwidth (Bw)

$$\Delta f = f_r - f_1$$

$$\Delta f = \frac{R}{2\pi L} \text{ (Hz)}$$

$$\Delta \omega = \frac{R}{L} \text{ (Rad/sec)}$$

* Fractional Band width

$$\frac{\Delta f}{f_r} = \frac{\Delta \omega}{\omega_r}$$

$$\frac{R/L}{1/\sqrt{2L}}$$

$$= \boxed{R\sqrt{\frac{C}{L}}}$$

* Quality factor (Q)

→ Measure of sharpness of 'I' v/s 'f' curve.

→ If 'Q' ↑ ⇒ sharpness ↑

Q ↓ ⇒ sharpness ↓

→ It is inverse of fractional Bandwidth.

$$\text{ii} \rightarrow Q = \frac{1}{f_r \cdot Bw} = \boxed{\frac{1}{R} \sqrt{\frac{L}{C}}}$$

$$\text{iii} \rightarrow Q = \frac{\omega_r}{\Delta \omega} = \frac{\omega_r}{R/L} = \frac{\omega_r L}{R} = \boxed{\frac{(X_L)_{\text{resonance}}}{R}}$$

$$Q = \frac{(X_L)_{\text{reso}}}{R} = \frac{(X_C)_{\text{reso}}}{R}$$

$$\text{iiii} \rightarrow Q = \frac{(V_L)_{\text{reso}}}{V_R} = \frac{(V_C)_{\text{reso}}}{V_R} = \frac{(V_L)_{\text{reso}}}{V_{\text{supply}}} = \frac{(V_C)_{\text{reso}}}{V_{\text{supply}}}$$

Power in AC ckt

$$\text{ii} \rightarrow P_{\text{inst}} = V_{\text{inst}} \cdot I_{\text{inst}}$$

$$\text{iii} \rightarrow P_{\text{peak}} = V_{\text{peak}} \cdot I_{\text{peak}}$$

$$\text{iii} \rightarrow P_{\text{apparent}} = V_{\text{rms}} \cdot I_{\text{rms}}$$

* Real/Average power of ckt.

$$P = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

$$P = I_{\text{rms}}^2 R = \left(\frac{V_R}{R}\right)^2 R$$

**

* ckt voltage & current

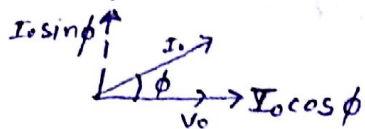
$$P = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

$$* P = I_{\text{rms}}^2 R = \frac{V_R^2}{R}$$

(Resistance voltage)

Wattless current / Workless current / Powerless current

It is part of ckt current which is not responsible for any power consumption.



$$\begin{aligned} * I_{\text{Wattless}} &= I_{\text{rms}} \sin \phi \\ * I_{\text{Wattfull}} &= I_{\text{rms}} \cos \phi \\ * I_{\text{ckt}} &= \text{Pythagorous of both} \end{aligned}$$

Skin Effect →

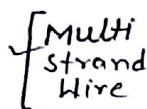
- * High freq. AC does flow near the surface of wire.
- * So, AC cable is made by multiple wire of smaller cross sectional area rather than thick cable.



$$R = \frac{\rho L}{A}$$

∴ A (↓↓↓) ⇒ Res. (↑↑↑)

$$\therefore \text{Power Loss} = I^2 R \text{ Loss} (\uparrow \uparrow)$$



$$A_{\text{eff}} (\uparrow) \Rightarrow R_{\text{eff}} (\downarrow)$$

$$P_{\text{Loss}} \Rightarrow \downarrow$$

Hot Wire Ammeter & Voltmeter

i) → Moving coil galvanometer does not work on AC ckt. since it's reversed direction continuously.

So, measure AC, A device is made based on heating effect of current called Hot wire Ammeter & Voltmeter.

* Its direction is direction independent.

$$\text{Deflection } \theta \propto H$$

$$\begin{aligned} \theta &\propto i_{\text{rms}}^2 \\ \theta &\propto V_{\text{rms}}^2 \end{aligned} \quad \begin{aligned} H &\propto i_{\text{rms}}^2 \\ H &\propto V_{\text{rms}}^2 \end{aligned}$$

ii) → It measure r.m.s value.

iii) → It has non-linear scale.

iv) → It can work in AC & DC Both.

NOTE → i) In case of Resonance in R-L-C ckt Amplitude of current is max.

For max Amplitude $X_C = X_L$

Resonance Angular Frequency.

$$\omega = \frac{1}{\sqrt{LC}}$$

Linear Frequency.

$$f = \frac{1}{2\pi\sqrt{LC}}$$

iii) → Avg power

$\cos \phi \rightarrow$ power factor.

$$P_{\text{avg}} = \frac{V_{\text{rms}}^2 \cdot R}{|Z|^2}$$

* pure 'R' ⇒ $\phi = 0$
 $\cos \phi = \max = 1$

* pure 'L' or 'C' ⇒ $\phi = \frac{\pi}{2}$ or $-\frac{\pi}{2}$
 $\cos \phi = 0$

(Wattless current found in ckt).

* In India	* In USA
220V	110V
50 Hz	60 Hz

* 1 unit = 1 kWh = $1000 \times 60 \times 60 = 3.6 \times 10^6$ Jule.
 * Any scalar quantity come with phase diff. in physics then vector addition occur. not simple addition.

- * R $\rightarrow$ power खाता है।
- * L $\rightarrow$ power वापस खाता है।
- * L-C $\rightarrow$ खसड़ा करते है।

** # Phase & Amplitude Relation for Alternating current & voltage.

SYMBOL	Impedance	Phase of current	Phase Angle (ϕ)	Amplitude Relation
		In phase $\bar{V}_R$	0°	$V_R = IR$
R	R			
		Leads V_C by 90°	-90°	$V_C = IX_C$
C	X_C			
		Lags V_L by 90°	$+90^\circ$	$V_L = IX_L$
L	X_L			

Mnemonic $\Rightarrow$ designed to assist the memory.
 $\hookrightarrow$ 'ELI the ICE man'

- * L $\Rightarrow$ Inductance
- * C $\Rightarrow$ capacitance
- * E $\Rightarrow$ voltage
- * I $\Rightarrow$ current.

- * In Inductive ckt (ELI) $\rightarrow$ the current (I) lags the voltage (E)
- * In capacitive ckt (ICE) $\rightarrow$ the current leads the voltage.